

A MMSE-based Beamforming Algorithm for MIMO Point-to-Point Full-Duplex Communication Systems

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Abstract— In this paper, we focus on the precoding design for sum rate maximization while considering the effects of residual SI for point – to-point multiple input/multiple output (MIMO) Full-Duplex systems. The MMSE-based beamforming algorithm was proposed to cancel the SI. The results shown that, the self-interference cancellation is done by matrix precoding at the transmitter if the total number of transmitting antenna of two nodes is greater than the number of receiving antenna of one node. The Bit Error Rate (BER) was also evaluated in the simulation.

Keywords— MIMO, full-duplex, self-interference, beamforming, precoding, postcoding, MMSE.

I. INTRODUCTION

The capacity of system became the one of the most important features of next generation wireless communications networks because of the explosive demand for data. Key techniques in advanced wireless networks improved the capacity by increasing spectrum efficiency and reducing latency. The most common radio transmission technique is half-duplex, in which the transmit and receive signals allocate two distinct frequency bands. The advantage of this method is that transmitting and receiving signals do not cause mutual interference, however, low spectrum utilization efficiency. In addition, another half-duplex type has also been applied, which is the transmit and receive signals allocated in two different time slots. In this type, interference is avoided but it is not efficient in terms of time [1].

Full-duplex transmission is considered a double effective technique compared to half-duplex. In this technique, the transmit and receive signals are transmitted in the same frequency at the same time. However, this technique exists self-interference (SI) signal from the transmitter antenna to the receiving antenna of the device itself. Therefore, full-duplex communication must conduct interference cancelation in various forms in different layers [2], [3].

Multi input / multiple output (MIMO) technology has been shown to have high spectral efficiency in wireless communication systems [4]. The works [5], [6] focus on two main problems in multi-antenna radio transmission systems, namely, capacity optimization and power control. Zero-

forcing coding is one of the underlying techniques for solving this problem [7]. Block-diagonal is a pre-coding technique that forces the matrix's diagonal interference signals to zero to eliminate these components.

Authors in [8] - [11] considered the pre-coding technique for full-duplex transmission using a transistor focused on self-interference signaling. Unlike these above works, this article focuses on point-to-point full-duplex communication, designs pre/post coding matrices to maximize system capacity.

The remaining parts of this paper are organized as follows. Section II delineates the system model of full-duplex MIMO-OFDM. The proposed beamforming matrixes design is developed in Section III. Analytical and empirical results with relevant discussions are located in Section IV. Finally, Section V provides some concluding remarks of this research work.

Notations: $\mathbf{X} \in \mathbb{C}^{r \times c}$ denote the complex matrix with r rows and c columns, \mathbf{X}^H is the transpose and conjugate transpose (Hermitian operator) of the matrix \mathbf{X} . $|\mathbf{X}|$ is the determinant of the matrix \mathbf{X} . $E(\cdot)$ stands for the expectation operator. $rank(\mathbf{X})$ and $trace(\mathbf{X})$ denote the rank and trace operator of the matrix \mathbf{X} , respectively. \mathbf{I}_y is the identity matrix having y rows. And $blkdiag(\mathbf{X}, \mathbf{Y})$ is block diagonal operation from the matrixes \mathbf{X} and \mathbf{Y} .

II. SYSTEM MODEL

In this paper, we consider a MIMO-OFDM full-duplex system as illustrated in Fig. 1, each node equips n_T transmit antennas and n_R receive antennas.

The received signal at node 1 and node 2 can be expressed by

$$\mathbf{r}_1 = \mathbf{H}_1 \mathbf{V}_1 \mathbf{x}_1 + \mathbf{G}_2 \mathbf{V}_2 \mathbf{x}_2 + \mathbf{z}_1, \quad (1)$$

$$\mathbf{r}_2 = \mathbf{H}_2 \mathbf{V}_2 \mathbf{x}_2 + \mathbf{G}_1 \mathbf{V}_1 \mathbf{x}_1 + \mathbf{z}_2, \quad (2)$$

where $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{C}^{n_T \times 1}$ are the transmitted signal vectors from node 1 and node 2, respectively; $\mathbf{V}_1, \mathbf{V}_2 \in \mathbb{C}^{n_T \times n_T}$ denote the precoding matrices at node 1 and node 2, respectively; $\mathbf{G}_1, \mathbf{G}_2 \in \mathbb{C}^{n_R \times n_T}$ stand for the matrices of channel gain from node 1 to node 2 and vice versa; $\mathbf{H}_1, \mathbf{H}_2 \in \mathbb{C}^{n_R \times n_T}$ are the self-interference matrices of channel gain at node 1 and node 2, respectively; $\mathbf{z}_1, \mathbf{z}_2 \in \mathbb{C}^{n_R \times 1}$ are additive white Gaussian noise (AWGN) with variance $\sigma_1^2 \mathbf{I}_R, \sigma_2^2 \mathbf{I}_R$ at node 1 and node 2, respectively.

In (1) and (2), the first component on the right side is the self-interfering signal, the second component on the right side is the desired signal. Pre/post-coding techniques are applied to eliminate this first component in full-duplex communication system.

The desired signal can be restored by multiplying the post coding matrix by the following:

$$\mathbf{y}_1 = \mathbf{T}_1 \mathbf{r}_1 = \mathbf{T}_1 (\mathbf{H}_1 \mathbf{V}_1 \mathbf{x}_1 + \mathbf{G}_2 \mathbf{V}_2 \mathbf{x}_2 + \mathbf{z}_1), \quad (3)$$

$$\mathbf{y}_2 = \mathbf{T}_2 \mathbf{r}_2 = \mathbf{T}_2 (\mathbf{H}_2 \mathbf{V}_2 \mathbf{x}_2 + \mathbf{G}_1 \mathbf{V}_1 \mathbf{x}_1 + \mathbf{z}_2). \quad (4)$$

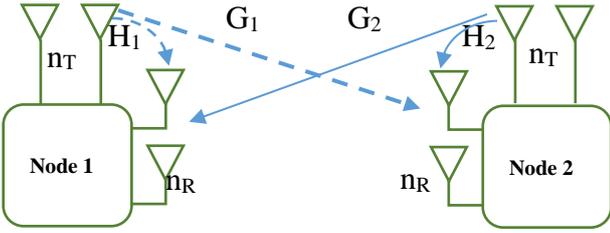


Fig. 1 System model of full-duplex communication system.

III. MMSE BEAMFORMING DESIGNS

Precoding matrices must be designed to eliminate the self-interference signal but still ensure the limited transmit power at each node ($\text{trace}(\mathbf{V}_i \mathbf{V}_i^H) \leq P, i = 1, 2$, where P is the transmit power constrain).

From (3) and (4), desired signal MSE at node i can be represented as

$$E_i = \mathbf{T}_i \mathbf{G}_j \mathbf{V}_j \mathbf{V}_j^H \mathbf{G}_j^H \mathbf{T}_i^H - \mathbf{T}_i \mathbf{G}_j \mathbf{V}_j - \mathbf{V}_j^H \mathbf{G}_j^H \mathbf{T}_i^H + \mathbf{I} + \mathbf{T}_i \mathbf{H}_i \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_i^H \mathbf{T}_i^H + \sigma^2 \mathbf{T}_i \mathbf{T}_i^H, \quad (5)$$

$$i = 1, 2, j = (i \bmod 2) + 1.$$

The desired signal MSE minimization under the per-node power constraint can be formulated as

$$\min \sum_{i=1}^2 \text{trace}(E_i), \quad (6)$$

$$\text{subject to: } \text{trace}(\mathbf{V}_i \mathbf{V}_i^H) \leq P. \quad (7)$$

To solve (7), we construct the Lagrangian function as

$$\mathcal{L}(\mathbf{T}_i, \mathbf{V}_i, \lambda_i) = \mathbf{T}_1 \mathbf{G}_2 \mathbf{V}_2 \mathbf{V}_2^H \mathbf{G}_2^H \mathbf{T}_1^H - \mathbf{T}_1 \mathbf{G}_2 \mathbf{V}_2 - \mathbf{V}_2^H \mathbf{G}_2^H \mathbf{T}_1^H + \mathbf{T}_2 \mathbf{G}_1 \mathbf{V}_1 \mathbf{V}_1^H \mathbf{G}_1^H \mathbf{T}_2^H - \mathbf{T}_2 \mathbf{G}_1 \mathbf{V}_1 - \mathbf{V}_1^H \mathbf{G}_1^H \mathbf{T}_2^H + 2\mathbf{I} + \lambda_1 (\text{trace}(\mathbf{V}_1 \mathbf{V}_1^H) - P) + \lambda_2 (\text{trace}(\mathbf{V}_2 \mathbf{V}_2^H) - P), \quad (8)$$

The Karush-Kuhn-Tucker (KKT) conditions [12] for (8) for $i=1, 2$ are given by

$$\frac{\partial \mathcal{L}(\mathbf{T}_i, \mathbf{V}_i, \lambda_i)}{\partial \mathbf{T}_i^*} = \mathbf{T}_i \mathbf{G}_j \mathbf{V}_j \mathbf{V}_j^H \mathbf{G}_j^H - \mathbf{V}_j^H \mathbf{G}_j^H + \mathbf{T}_i \mathbf{H}_i \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_i^H + \sigma^2 \mathbf{T}_i = 0, \quad (9)$$

$$\frac{\partial \mathcal{L}(\mathbf{T}_i, \mathbf{V}_i, \lambda_i)}{\partial \mathbf{V}_i^*} = \mathbf{G}_i \mathbf{T}_j \mathbf{T}_j^H \mathbf{G}_i^H \mathbf{V}_i^H - \mathbf{G}_i \mathbf{T}_j^H + \mathbf{H}_i^H \mathbf{T}_i^H \mathbf{T}_i \mathbf{H}_i \mathbf{V}_i + \lambda_i \mathbf{V}_i = 0, \quad (10)$$

$$\frac{\partial \mathcal{L}(\mathbf{T}_i, \mathbf{V}_i, \lambda_i)}{\partial \lambda_i} = \text{trace}(\mathbf{V}_i \mathbf{V}_i^H) - P = 0. \quad (11)$$

Finally, the optimal pre/postcoding matrix are computed as

$$\mathbf{T}_i = \mathbf{V}_j^H \mathbf{G}_j^H (\mathbf{G}_j \mathbf{V}_j \mathbf{V}_j^H \mathbf{G}_j^H + \mathbf{H}_i \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_i^H + \sigma^2 \mathbf{I})^{-1}, \quad (12)$$

$$\mathbf{V}_i = (\mathbf{G}_i^H \mathbf{V}_j^H \mathbf{G}_j \mathbf{G}_j + \mathbf{H}_i \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_i^H + \lambda_i \mathbf{I})^{-1} \mathbf{G}_i \mathbf{T}_j, \quad (13)$$

where λ_i satisfying the individual power constraint in (11) (with \mathbf{V}_i is given) can be found by bisection root method.

The overall proposed MSE minimization scheme is summarized below:

ALGORITHM 1. MSE-BASED BEAMFORMING DESIGN FOR MIMO FULL-DUPLEX COMMUNICATION SYSTEM

Initialization:

- Choose an initial value \mathbf{V}_i satisfying the power constraint in (11),

While (not converge)

- For fixed \mathbf{V}_i , calculate \mathbf{T}_i according to (12).
- For found \mathbf{T}_i , calculate λ_i according to (11).
- Update \mathbf{V}_i according to (13).

end

IV. SIMULATION RESULTS

In this section, computer simulations are conducted to provide numerical results for illustrating the performance of proposed beamforming algorithms. In the figures of numerical results, each plotted point of the system performance is obtained by averaging over 1000 independent channel realizations.

In the simulated system, signal power is normalized and the system SNR is defined as $\frac{1}{\sigma^2}$. In generating channel responses, elements of $\mathbf{H}_i, \mathbf{G}_i$ are assumed to be independently random complex values following $\mathcal{N}(0,1)$. The channel response is assumed to be a block-fading. Modulation types include BPSK, QPSK, 16-QAM and 64-QAM.

TABLE 1

THE SELF-INTERFERENCE CHANNEL MATRIX IN NODE 1 (H1)

-1.141-0.2877i	0.3114 +0.7959i	-0.2050+0.1065i	0.8151+0.9146i
-0.5564-0.2107i	1.0281+0.9919i	0.9278+0.0815i	0.8910+0.8768i

TABLE 2

THE PRECODING MATRIX IN NODE 1 (V1)

0.7069+0.0000i	-0.0004-0.0004i
-0.0004+0.0004i	0.7074 -0.0000i

TABLE 3
THE POSTCODING MATRIX IN NODE 1 (T_1)

0.0344 - 0.5331i	0.5592 + 0.0158i
-0.2918 - 0.5749i	-0.6127 - 0.0062i
0.0200 + 0.6798i	0.0410 + 0.1043i
-0.0823 + 0.0330i	0.8737- 0.1396i

TABLE 4
THE DESIRED-SIGNAL CHANNEL MATRIX IN NODE 1 (G_2)

-0.1713+1.2413i	-0.5679+1.3452i	-0.0835+0.2912i	-0.2542+0.1041i
0.3325+0.0235i	-0.3507-0.8347i	-0.4733-0.6575i	1.1609-0.3360i

TABLE 5
THE PRECODING MATRIX IN NODE 2 (V_2)

0.7070 - 0.0000i	0.0001 - 0.0001i
0.0001 + 0.0001i	0.7072 + 0.0000i

TABLE 6
THE SELF-INTERFERENCE BEAMFORMING MATRICES IN NODE 1 ($T_1 H_1 V_1$)

0.0002 + 0.0011i	0.0007 - 0.0001i
-0.0003 - 0.0011i	-0.0008 + 0.0008i

Tables 1-7 are matrices in the point-to-point full-duplex communication system with the antenna configuration ($N_R = 2, N_T = 4$). In this tables, the channel matrices are randomly generated in loops. From these matrices, we find the beamforming matrices in formulas (12) and (13), then check whether constraints (7). Tables 1, 2 and 3 represent the matrices at node 1, including self-interference channel matrix H_1 , the precoding matrix V_1 and the post coding matrix T_1 . Tables 4 and 5 represent the matrices at node 2, including the desired signal channel matrix G_2 and the precoding matrix V_2 . In Table 6, we introduce the self-interference beamforming matrices in node 1 ($T_1 H_1 V_1$). We can see that the self-interfering signal is almost eliminated. In addition, in Table 7, we introduce the desired signal beamforming matrices in node 1 ($T_1 G_2 V_2$). The result show that $T_1 G_2 V_2 \sim I$, so the desired signal completely restored.

TABLE 7
THE DESIRED-SIGNAL BEAMFORMING MATRICES IN NODE 1 ($T_1 G_2 V_2$)

0.9988 - .0000i	-.0002+ 0.0001i
-0.0002- 0.0001i	0.9985 + 0.0000i

Fig. 2 shows the average sum-rate of the two systems with the antenna configuration ($N_R = 4, N_T = 8$) and ($N_R = 2, N_T = 4$), respectively. The result show that the higher antenna configuration provides the greater signal speed.

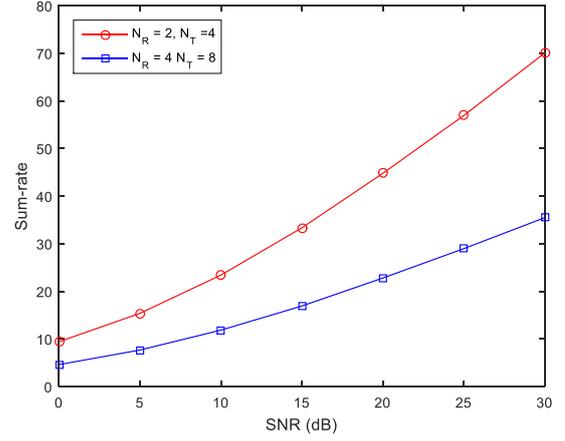


Fig. 2 Full-duplex beamforming Sum-rate versus SNR with difference antenna configurations

Fig. 3 shows the MSE vs SNR of the two systems with the antenna configuration ($N_R = 4, N_T = 8$) and ($N_R = 2, N_T = 4$), respectively. The result show that the higher antenna configuration will give the better performance.

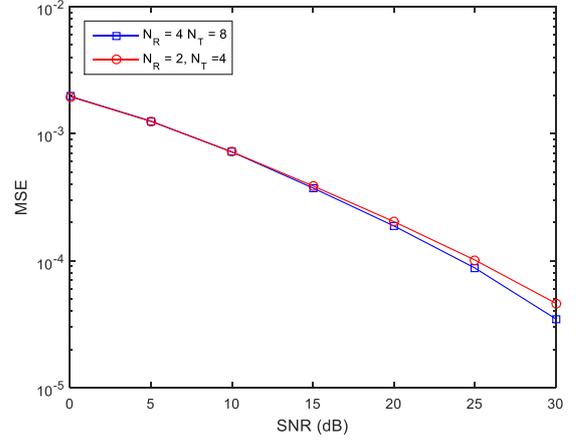


Fig. 3 Full-duplex beamforming MSE versus SNR with difference antenna configurations.

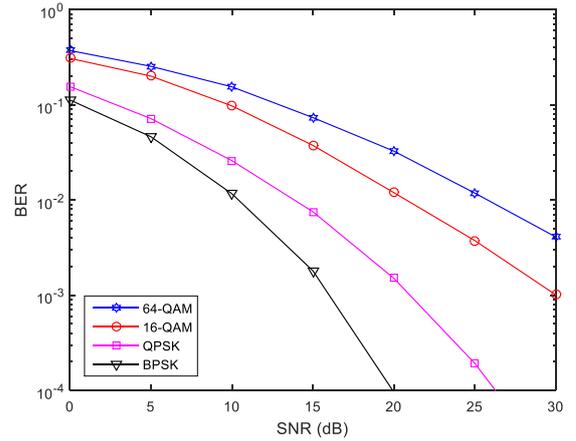


Fig. 4 Full-duplex beamforming BER versus SNR with the antenna configuration ($N_R = 4, N_T = 8$).

Fig. 4 shows bit error rate (BER) versus SNR in the with modulation types BPSK, QPSK, 16-QAM and 64-QAM, respectively. In this configuration, each node equipped $N_R = 4$ receive antennas and $N_T = 8$ transmit antennas. We can see that, BPSK gives the best results and 64-QAM gives the worst result. As many constellations are more susceptible to interference and interference.

Fig. 2, 3 and 4 show that the proposed algorithm full-duplex beamforming provides good performance because the self-interference almost eliminated completely.

V. CONCLUSION

This paper considered the problem of MIMO full-duplex bidirectional communication between a pair of MIMO modems in the presence of noise and self-interference. We proposed a beamforming scheme base on minimizing total mean square errors under the individual transmit power constraints. From the results obtained, it was found that the proposed algorithm full-duplex beamforming provides good performance because the self-interference almost eliminated completely.

NOMENCLATURE

BER	Bit Error Rate
BPSK	Binary phase-shift keying
DS	Desire Signal
FD	Full-Duplex
MIMO	Multiple input/multiple output
MMSE	Minimum mean square error
QAM	Quadrature amplitude modulation
QPSK	Quadrature phase-shift keying
SI	Self-Interference

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